Solving Quadratic Equations with Quadratic Formula

f(x) = ax^2 + bx + c

$$Quadatic\ Formula\ is\ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic Formula can be used to solve for any quadratic equation. When solving quadratic equations, we are finding the roots/zeros for the quadratic functions.

Examples

$f(x) = x^2 + 6x - 16$

What is/are the root(s)/zero(s) for this function?

1. $x^2 + 6x - 16 = 0$
2. a = 1, b = 6, c = -16
3. $x = \frac{-(6)\pm\sqrt{(6)^2-4(1)(-16)}}{2(1)}$
4. $x = \frac{-(6)\pm\sqrt{36-(64)}}{2(1)}$

$$x = \frac{-(6) \pm \sqrt{100}}{2}$$

$x = \frac{-6\pm10}{2}$, which mean

$x = \frac{-6-10}{2} = -\frac{16}{2} = -8$

and

$x = \frac{-6+10}{2} = \frac{4}{2} = 2$

x = -8, 2

$6x^2 - x = 2$

Find the root(s) for this equation?

1. $6x^2 - x - 2 = 0$
2. a = 6, b = -1, c = -2
3. $x = \frac{-(-1)\pm\sqrt{(-1)^2-4(6)(-2)}}{2(6)}$
4. $x = \frac{1\pm\sqrt{1-(-48)}}{12}$

$$x = \frac{1 \pm \sqrt{49}}{12}$$

x $= \frac{1\pm7}{12}$, which mean

$x = \frac{1+7}{12} = \frac{8}{12} = \frac{2}{3}$

and

$x = \frac{1-7}{12} = -\frac{6}{12} = -\frac{1}{2}$

$x = -\frac{1}{2}, \quad \frac{2}{3}$

Steps

Solving Quadratic Equation

f(x) = ax^2 + bx + c

1. Set the equation = 0, in the standard form ax^2 + bx + c = 0.
2. Identify the values for *a*, *b* and *c*.
3. Replace the variables with the corresponding values in the formula.
4. Simplify the solution(s).

Practice Problems

$g(x) = x^2 - 11x + 28$

$9x^2 = -12x - 4$

$-12x - 4 = 9x^2$

<table>
<tr>
<td>$2x^2 - 4x + 5 = 0$
What is/are the root(s) for the equation?
1. Rewrite the equation because it is already set equal to 0.
$2x^2 - 4x + 5 = 0$
2. a = 2, b = -4, c = 5
3. $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(5)}}{2(2)}$
4. $x = \frac{4 \pm \sqrt{16-40}}{4}$
$x = \frac{4 \pm \sqrt{-24}}{4}$
$x = \frac{4 \pm \sqrt{24i}}{4}$
$x = \frac{4 \pm 2i\sqrt{6}}{4}$
$x = \frac{2 \pm (\sqrt{6})i}{2}$</td>
<td>Notes
-24 under the radical ($\sqrt{}$) can be written as $\sqrt{24i}$, which will allow us to simplify the solution. This type of solution is known as complex/imaginary roots</td>
<td>$10x^2 = -5x - 1$</td>
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<td colspan="3">When solving quadratic equations with quadratic formula, you may obtain 2 real roots/zeros, 2 complex/imaginary roots or just one root. The type of roots/zeros can be determined by calculating the discriminant, D.
Discriminant, D = $b^2 - 4ac$
When D > 0, you will have 2 real roots/zeros.
When D < 0, you will have 2 complex/imaginary roots
When D = 0, you will have only 1 root/zero.

For the equation $2x^2 - 4x + 5 = 0$, the discriminant is less than 0 (-24 < 0); therefore we have 2 complex roots.</td>
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<td>Use the discriminant to determine the type of solutions for the quadratic equation $4x^2 + 12x = -9$ and then find the root(s).

1. $4x^2 + 12x + 9 = 0$
2. a = 4, b = 12, c = 9
3. $D = 12^2 - 4(4)(9)$
4. $D = 144 - 144 = 0$
5. This equation has 1 real solution: root/zero

To find the root(s), use the quadratic formula.
$x = \frac{-(12) \pm \sqrt{0}}{2(4)}$
$x = \frac{-12}{8} = -\frac{3}{2}$</td>
<td>Calculating the discriminant
$D = b^2 - 4ac$
1. Set the equation equal 0 in standard form $ax^2 + bx + c = 0$.
2. Identify the values for a, b, and c.
3. Substitute the variables with the corresponding values.
4. Simplify
5. Draw your conclusion.</td>
<td>$9x^2 = 30x - 25$

$x^2 + 1 = -x$</td>
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