Solving Quadratic Equations by Completing the Square

$f(x) = ax^2 + bx + c$

When solving for quadratic equations/functions, we are finding their roots/zeros. Some of the equations/functions are not factorable, we can use complete the square method. Complete the square creating perfect square trinomial expression is $x^2 + bx + (\frac{b}{2})^2$ which can be expressed as binomial square $(x + \frac{b}{2})^2$.

Examples

a. $x^2 + 6x$

1. $x^2 + 6x + (\frac{6}{2})^2$
2. $(x + 3)^2$

b. $x^2 - 12x$

1. $x^2 - 12x + (-\frac{12}{2})^2$
2. $(x + (-6))^2 = (x - 6)^2$

c. $x^2 - \frac{21}{4}x$

1. $x^2 - \frac{21}{4}x + (-\frac{21}{8})^2$
2. $(x - \frac{21}{8})^2$

Steps

Complete the Square

1. Take half of b, square it, and add to the expression.
2. Write the perfect square trinomial expression in term of binomial square.

Practice Problems

1. $x^2 + 8x$ + ________?
2. $x^2 - 14x$ + _______?
3. $x^2 - \frac{13}{3}x$ + ______?

Complete the square for an expression, simply take half b value, square, add it to the expression, and then write the expression in term of binomial square. Completing the square for solving quadratic equations, we must preserve the property of equality. This mean, if we have to add $(\frac{b}{2})^2$ to both sides of the equation.

x^2 + ___x + 25 x^2 − ____x + 36 x^2 ___ ___x + (- ___)^2 x^2 ___ ___x + (___)^2

$f(x) = x^2 - 6x + 7$

1. $x^2 - 6x + 7$
2. $x^2 - 6x = -7$
3. a = 1, skip this step.
4. $x^2 - 6x + (-3)^2 = -7 + (-3)^2$
5. $(x-3)^2 = 2$
6. $\sqrt{(x - 3)^2} = \sqrt{2}$

   $x - 3 = \pm\sqrt{2}$

   $x = 3 \pm\sqrt{2}$

Solving Quadratic Equation by Completing the Square

1. Set the equation/function equal to 0 in standard form, $ax^2 + bx + c = 0$.
2. Collect variables to 1 side of the equation and constant/integer on the other side.
3. If needed, make a = 1.
4. Add $(\frac{b}{2})^2$ to both sides of the equation.
5. Write the perfect square trinomials as binomial square.
6. Solve for the variable.

$f(x) = x^2 + 4x - 6$

<table>
<tr>
<td>$3x^2 + 5x = 2$
1. Rewrite the equation because the variables are already on 1 side of the equation and the constant is on the other side, $3x^2 + 5x = 2$ and skip to step 3.
3. $\frac{3x^2 + 5x}{3} = \frac{2}{3}$
4. $x^2 + \frac{5}{3}x + (\frac{5}{6})^2 = \frac{2}{3} + (\frac{5}{6})^2$
5. $(x + \frac{5}{6})^2 = \frac{2}{3} + \frac{25}{36}$
6. $\sqrt{(x + \frac{5}{6})^2} = \sqrt{\frac{49}{36}}$
$x + \frac{5}{6} = \pm\frac{7}{6}$
$x = \pm\frac{7}{6} - \frac{5}{6}$, which mean
$x = -\frac{7}{6} - \frac{5}{6} = -\frac{12}{6} = -2$
and
$x = \frac{7}{6} - \frac{5}{6} = \frac{2}{6} = \frac{1}{3}$
$x = -2, \frac{1}{3}$

$2x^2 - x - 2 = 0$
1. Rewrite the equation
2. $2x^2 - x = 2$
3. $\frac{2x^2 - x}{2} = \frac{2}{2}$
4. $x^2 - x + (-\frac{1}{2})^2 = 1 + (-\frac{1}{2})^2$
5. $(x - \frac{1}{2})^2 = 1 + \frac{1}{4}$
6. $\sqrt{(x - \frac{1}{2})^2} = \sqrt{\frac{5}{4}}$
$x - \frac{1}{2} = \pm\frac{\sqrt{5}}{\sqrt{4}}$
$x = \frac{1}{2} \pm \frac{\sqrt{5}}{2} = \frac{1 \pm \sqrt{5}}{2}$</td>
<td></td>
<td>$4x^2 + 16x = 128$

$3x^2 - 24x - 5 = 0$</td>
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<tr>
<td colspan="3">Complete the Square method creates vertex form of equation, $f(x) = a(x + \mathbf{h})^2 + \mathbf{k}$, and the vertex is $(\mathbf{h}, \mathbf{k})$. Vertex is the lowest or highest point on the parabola. The vertex can be found from $f(x) = ax^2 + bx + c$ by using the formula $-\frac{b}{2a}$. The vertex is $(-\frac{b}{2a}, f(-\frac{b}{2a}))$. $(-\frac{b}{2a})$ is an axis of symmetry, a vertical line split the graph into 2 equal halves.</td>
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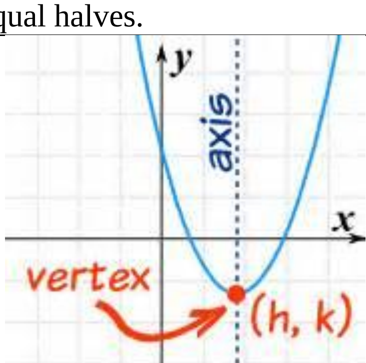
<td>
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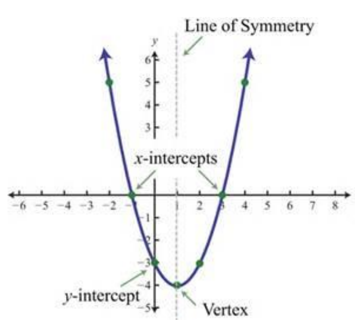
<td>
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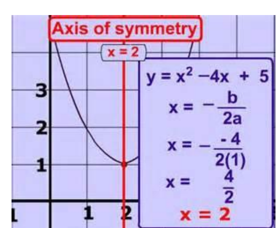
<td>

$f(2) = (2)^2 - 4(2) + 5 = 1$
Vertex (2, 1)</td>
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