

Rate of Change

Rate of change is often used when speaking about momentum. It is generally expressed as a ratio between a change in one variable relative to a corresponding change in another. Rate of change (average rate of change) is constant. Algebraically, the rate of change is represented by the slope of a line.

$$\text{Slope, } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{Changes in } y}{\text{Changes in } x} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$$

Example 1

A climber is on a hike. After 2 hours he is at an altitude of 400 feet. After 6 hours, he is at an altitude of 700 feet. What is the average rate of change?

- Change in altitude over time.
(2, 400) and (6, 700)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{700 - 400}{6 - 2}$$

$$m = \frac{300}{4} = 75$$

Average rate of change is 75 feet per hour.

- $y = mx + b$
- $y - y_1 = m(x - x_1)$
- $y - 400 = 75(x - 2)$
- $y - 400 = 75x - 150$
- $y = 75x + 250$

Example 2

A scuba diver is 30 feet below the surface of the water 10 seconds after he entered the water and 100 feet below the surface after 40 seconds. What is the scuba divers rate of change?

Solution:

1. Identify the type of change and record the values.
2. Apply the slope formula and calculate the rate of change.
3. If necessary, create a linear equation by using point slope formula to predict future value.

1. A rocket is 1 mile above the earth in 30 seconds and 5 miles above the earth in 2.5 minutes.

What is the rockets rate of change in miles per second? What about miles per minute.

2. Michael started a savings account with \$300. After 4 weeks, he had \$350 dollars, and after 9 weeks, he had \$400. What is the rate of change of money in his savings account per week?

Example 3

Find the Average Rate of Change from a Table

The table below defines the relationship $y = f(x)$

| | | | | |
|------|----|----|---|---|
| x | 0 | 2 | 4 | 5 |
| f(x) | 26 | 17 | 5 | 1 |

Find the average rate of change of f with respect to x over [0, 4].

$$\frac{\text{change of } f}{\text{change of } x} = \frac{f(4) - f(0)}{4 - 0}$$

$$\frac{5 - 26}{4 - 0} = -\frac{21}{4}$$

[Average rate of Change Video](#)

When you have a constant rate of change at every interval, it is called a common difference of an arithmetic sequence. It is called common difference because it is the same, or common to, each number and it also is the difference between each number in the sequence.

Example 4

| | | |
|----|----|--------------------------|
| x | y | } difference of y-values |
| -2 | -4 | |
| -1 | -1 | |
| 0 | 2 | |
| 1 | 5 | |
| 2 | 8 | |

$-1 + 4 = 3$
 $2 + 1 = 3$
 $5 - 2 = 3$
 $8 - 5 = 3$

Example 5

| Temperature | Ice Cream Sales |
|-------------|-----------------|
| 85 | \$200 |
| 71 | \$160 |
| 84 | \$170 |
| 66 | \$120 |
| 77 | \$120 |
| 75 | \$180 |
| 68 | \$100 |
| 91 | \$230 |

Find the change in the numerator and denominator over [0, 4]

Put in the function values from the table

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3. The average price for a ticket to a movie theater in North America for selected years is shown in the table below.

| | | | | | | |
|-------|------|------|------|------|------|------|
| Year | 1987 | 1991 | 1995 | 1999 | 2003 | 2007 |
| Price | 3.91 | 4.21 | 4.35 | 5.06 | 6.03 | 6.88 |

Find the change over the interval [2, 5]

4. FUNDRAISING - The table shows the amount of money a Booster Club made washing cars for a fundraiser. Use the information to find the rate of change in dollars per car.

| Cars Washed | |
|-------------|------------|
| Number | Money (\$) |
| 5 | 40 |
| 10 | 80 |
| 15 | 120 |
| 20 | 160 |

Determine the rate of change per car.

5. Find the common difference from these tables of value if any.

| | |
|---|----|
| x | y |
| 0 | 3 |
| 1 | 8 |
| 2 | 13 |
| 3 | 18 |
| 4 | 23 |

| | |
|---|-------|
| x | y |
| 0 | 3 |
| 1 | 15 |
| 2 | 75 |
| 3 | 375 |
| 4 | 1,875 |