# Solving Radical Equations with a Single Variable.

Radical equations contain the radical (square root) sign (symbol) and/or fractional exponents. The strategy for solving radical equations is to get rid of the radicals. Radical expression can be expressed in term of exponent.

$$\sqrt{u} = u^{\frac{1}{2}}, \text{ so for } \sqrt{u-1} = (u-1)^{\frac{1}{2}}$$
$$\sqrt[3]{u} = u^{\frac{1}{3}}$$
$$\sqrt[4]{u} = u^{\frac{1}{4}}$$

## Example Problems

**Example 1**

$\sqrt{x} - 8 = 0$

1. $\sqrt{x} = 8$
2. Radical is square root.
3. $(\sqrt{x})^2 = 8^2$
4. No simplification requires.
5. x = 64
6. $\sqrt{64}$ - 8 = 0?
   8 – 8 = 0 ☺

**Example 2**

$2\sqrt{x+1} = 14$

1. $\sqrt{x+1} = \frac{14}{2} = 7$
2. Radical is square root.
3. $(\sqrt{x+1})^2 = 7^2$
4. x + 1 = 49
5. x = 48
6. $2\sqrt{48+1}$ = 14?
   $2\sqrt{49}$ = 14?
   $2 * 7 = 14$ ☺

**Example 3**

$5\sqrt[3]{4x+3} = 15$

1. $\sqrt[3]{4x+3} = \frac{15}{5} = 3$
2. Radical is cube root
3. $(\sqrt[3]{4x+3})^3 = 3^3$
4. 4x + 3 = 27
5. 4x = 24
   x = 6
6. $5\sqrt[3]{4(6)+3} = 15?$
   $5\sqrt[3]{27} = 15?$
   $5(3) = 15$ ☺

## Steps

**Solving Radical Equations**

1. Isolate radical
2. Identify the radical.
3. Raise both sides of an equation to the ***n*th** power.
4. Simplify by using algebraic operations.
5. Solve for the variable.
6. Check your answer by substituting.

## Practice Problems

**Practice 1**

$\sqrt{2x+3} = 4$

**Practice 2**

$-3\sqrt{2x+1} = -15$

**Practice 3**: Create your own radical equation and solve it.

| | | |
|---|---|---|
| **Example 4**<br>$(6b)^{\frac{1}{2}} = (8-2b)^{\frac{1}{2}}$<br>1. $\sqrt{6b} = \sqrt{8-2b}$<br>2. $(\sqrt{6b})^2 = (\sqrt{8-2b})^2$<br>3. $6b = 8 - 2b$<br>$8b = 8$<br>$b = 1$ | 1. Rewrite the equation in radical form.<br>2. Raise both sides of an equation to the ***n*th** power.<br>3. | **Practice 4**<br>$\sqrt{2m-6} = \sqrt{3m-14}$ |
| **Example 5**<br>$(20-r)^{\frac{1}{2}} = r$<br>1. $\sqrt{20-r} = r$<br>2. $(\sqrt{20-r})^2 = (r)^2$<br>3. $20 - r = r^2$<br>4. $0 = r^2 + r - 20$<br>5. $0 = (r+5)(r-4)$<br>6. $r = \{-5, 4\}$ | Explain what happened in step 4 onward. | **Practice 5**<br>$\sqrt{56-r} = r$ |
| **Example 6**<br>$\sqrt{2v-7} = v-3$<br>1. $(\sqrt{2v-7})^2 = (v-3)^2$<br>2. $2v - 7 = v^2 - 6v + 9$<br>3. $0 = v^2 + 8v + 16$<br>4. $0 = (v+4)(v+4)$<br>5. $0 = (v+4)^2$<br>6. $v = -4$ | | **Practice 6**<br>$x = 5 + (3x-11)^{\frac{1}{2}}$ |

When solving radical equations, you may encounter extraneous solution. An extraneous solution is a solution that does not satisfy the original equations, and therefore ***must not*** be listed as an actual solution.

**Example 1.** $\sqrt{3x} + 6 = 0$

1. $\sqrt{3x} = -6$
2. 3x = 36
3. x = 12
4. $\sqrt{3(12)} = -6$? **EXTRANEOUS SOLUTION!**

**Example 2.** $\sqrt[4]{3x} - \sqrt[4]{2x-5} = 0$