**Applications of College Algebra**

**Chapter 8D – Logarithmic Scales:** **Earthquakes, Sounds, and Acids**

**The Magnitude Scale for Earthquakes**

Scientists measure the strength of the earthquake with the magnitude scale (0-10). Each magnitude represents about 32 times as much energy as the previous magnitude.

Log10E = 4.4 + 1.5M or E = (2.5$ ∙ $104)$ ∙ $101.5M

**Example 1 The Meaning of One Magnitude**

Using the formula for earthquake magnitudes, calculate precisely how much more energy is released for each 1 magnitude on the earthquake scale.

 E = (2.5$ ∙ $104)$ ∙ $101.5M

 101.5(1) $≈$ 31.623 times

E = (2.5$ ∙ $104)$ ∙ $101.5(1) $≈$ 790,569 joules

How many times as much energy is released by an earthquake of magnitude 6 as by 1 magnitude of 4?

E = (2.5$ ∙ $104)$ ∙ $101.5(4) $≈$ 2.5x1010 joules

 E = (2.5$ ∙ $104)$ ∙ $101.5(6) $≈$ 2.5x1013 joules

**Example 2 Comparing Disasters**

In 1989 San Francisco earthquake, in which killed 90 people, has magnitude 7.1. Calculate the energy released, in joules. Compare the energy of this earthquake to that of the 2003 earthquake that destroyed the ancient city of Bam, Iran, which had magnitude 6.3 and killed an estimated 50,000 people.

E = (2.5$ ∙ $104)$ ∙ $101.5(7.1) $≈$ 1.1x1015 joules

The difference in magnitude: 7.1 – 6.3 = 0.8 greater

So, the energy was released: 101.5(0.8) $≈$ 16 times as much.

**Measuring Sounds in Decibels (dB)**

The decibel scale is used to compare the loudness of sounds. The softest sound audible to the human ears is 0 dB.

Loudness in dB = 10$∙$log10 ($\frac{intensity of the sound}{intensity of softest audible sound}$)

Or

$\frac{intensity of the sound}{intensity of softest audible sound}$ = 10(loudness in dB)/10

**Example 3 Computing Decibels**

Suppose a sound is 100 times as intense as the softest audible sound. What is its loudness, in decibels?

Loudness in dB = 10$∙$log10100 = 10x2 = 20 dB

So, the sound that is 100 times as intense as the softest audible sound has a loudness of 20 dB, which is equivalent to a whisper.

How many times as loud as the softest audible sound is the sound of a siren at 30 meters?

What is the loudness, in decibels, of a sound 10 trillion times as loud as the softest audible sound?

Loudness in dB = 10$∙$log10 ($10,000,000,000,000)$ = 130 dB

Suppose that a sound is 100 times as loud as (more intense than) a whisper. What is its loudness in decibels?

Loudness in dB = 10$∙$log10 ($\frac{100x100}{100})$ = 10$∙$log10 ($10,000)$ = 10x4 = 40 dB

**Example 4 Sound Comparison**

How does the intensity of a 57-dB sound compare to that o a 23-dB sound?

 $\frac{intensity of the sound}{intensity of softest audible sound}$ = 10(loudness in dB)/10

 $\frac{intensity of Sound 1}{intensity of Sound 2}$ = 10[(loudness of Sound 1 in dB) – (loudness of Sound 2 in dB)]/10

 $\frac{intensity of Sound 1}{intensity of Sound 2}$ = 10(57 – 23)/10 $≈$ 2512

 So, a sound of 57-dB is about 2500 times as intense as a sound of 23 dB.

How much louder (more intense) is a 35-dB sound than a 10-dB sound?

 $\frac{intensity of Sound 1}{intensity of Sound 2}$ = 10[(loudness of Sound 1 in dB) – (loudness of Sound 2 in dB)]/10

 $\frac{intensity of Sound 1}{intensity of Sound 2}$ = 10(35 - 10)/10 $≈$ 316 times more intense.

**The pH Scale for Acidity**

The pH is used by chemists to classify substances as neutral, acidic, or basic (alkaline).

Pure water is neutral and has a pH of 7.

Acids have a pH lower than 7.

Bases have a pH above 7.

pH = - log10[H+] or [H+] = 10-pH

[H+] is the hydrogen ion concentration in moles per liter.

**Example 6 Finding pH**

What is the pH of a solution with a hydrogen ion concentration of 10-12 mole per liter? Is it an acid or a base?

pH = - log10[10-12] = 12, it is a strong base.

If the pH of a solution decreases by 3.5 (e.g., from 6.5 to 3), how much does the hydrogen ion concentration change? Does the change make the solution more acidic or more basic?

 [H+] increases 103.5 moles per liter

 pH = - log10[10-3.5] = 3, which is more acidic.

What is the hydrogen ion concentration of a solution with pH 3.5?

[H+] = 10-3.5 = 3.16x10-4

What is the pH of a solution with a hydrogen ion concentration of 10-6 mole per liter? Is this solution an acid or a base?

 pH = - log10[10-6] = 6, slightly acid.

**Homework: 8D Exercise # 5, 7, 9, 11, 15, 17, 19, 25, 27, 29, 31**

**Homework 8C and 8D are due on Wednesday 1/27**

**Quiz on Chapter 8C and 8D on Wednesday 1/27**