**Chapter 8B – Doubling Time and Half-Life**

**Doubling Time**

The time required for each doubling in exponential growth is called **doubling time.** Consider an initial population of 10,000 that grows with a doubling time of 10 years.

In 10 years or in one doubling time, the population increased by a factor of 2, the new population would be 2 x 10,000 = 20,000.

In 20 years or in two doubling time, the population increased by a factor of 2x2 = 22 = 4, the new population would be 4 x 10,000 = 40,000.

After a time *t*, an exponential growing quantity with a doubling time of *Tdouble* increases in size by a factor of$ 2^{\frac{t}{T}}$. The new value of the growing quantity is related to its initial value (at *t* = 0) by

$new value=initial value$ x $2^{\frac{t}{T}}$

**Example 1 Doubling with Compound Interest**

Suppose your bank account has a doubling time of 13 years. By what factor does your balance increase in 50 years?

$2^{\frac{50}{13}}$ $≈14.38$

If you start with a balance of $1000, in 50 years it will grow to

$new value=1,000 x 2^{14.38}$ = $14,382

**Example 2 World Population Growth**

The world population doubled from 3 billion in 1960 to 6 billion in 2000. Suppose that the world population continued to grow (from 2000 on) with a doubling time of 40 years. What would the population be in 2030? in 2200?

**Rule of 70: Approximate Doubling Time Formula**

For a quantity growing exponentially at a rate of P% per period, the doubling time is approximately

*Tdouble* $≈ \frac{70}{P}$

The approximation works best for small growth rates and breaks down for growth rates over about 15%

**Example 3 Population Doubling Time**

The world population was about 6 billion in 2000 and was growing at a rate of about 1.4% per year. What is the approximate doubling time at the growth rate? If this growth rate were to continue, what would the world population be in 2030?

**Example 4 Solving the Doubling Time Formula**

The world population doubled in the 40 years from 1960 to 2000. What is the average percentage growth during this period? Contrast this growth with the 2000 growth rate of 1.4% per year.

**Exponential Decay and Half-Life**

Exponential decay occurs whenever a quantity decreases by the same percentage in every fixed time period. In the case, the value of the quantity repeatedly decreases to half its value, with each having occurring in a time called the half-live.

$new value=initial value x (.5)^{\frac{t}{T}}$ ; where *Thalf*

**Example 5 Carbon-14 Decay**

Radioactive carbon-14 has a half-live of about 5700 years. It collects in organisms only while they are alive. Once they are dead, it only decays. What fraction of the carbon-14 in an animal bone still remains 1000 years after the animal has died?

**Example 6 Plutonium after 100,000 Years**

Suppose 100 pounds of radioactive plutonium (Pu-239, has a half-live of about 24,000 years) is deposited at a nuclear waste site. How much it will still be present in 100,000 years?

**The Approximate Half-Life Formula**

For a quantity decaying exponentially at a rate of P% per period, the half-live is approximately

*Thalf* $≈ \frac{70}{P}$

The approximation works best for small decay rates and breaks down for decay rates over about 15%.

Exact Formulas for Doubling Time and Half-Life

For more precise work or for cases larger rates when the approximate formulas break down, we need the exact formulas.

*Tdouble* = $\frac{log\_{10}2}{log\_{10}(1+r)}$

*Thalf* = $- \frac{log\_{10}2}{log\_{10}(1+r)}$

Homework: Exercise 8B # 1 – 55 every other odd.

**Homework for 8A and 8A are due on Wednesday 1/13/15**