**Chapter 7: Unit E – Counting and Probability**

From 7A and 7A, computing theoretical probabilities requires counting outcomes. In this unit, you will learn counting tools that will enable us to solve a much wider range of probability.

**Arrangements with Repetition**

Suppose that license plate in any state can be display seven numerals. How many different 7-number license plates are possible? There are 10 possibilities (0-9) for the first position and the same 10 possibilities for the second position, and so on. The possibilities for the first 2 positions are 10x10, which are 100 possibilities. So, 7 positions, it would be 10x10x10x10x10x10x10 = 107 (10 million) possible arrangements of license plate. This type of counting problem, in which we repeatedly select from the same group of choices is called arrangement with repetition.

If we make *r* selections from a group of *n* choices, a total of *n* x *n* x *n* …x *n* = *nr* different arrangements are possible.

Example 1

How many seven-symbol license plates are possible if both numerals and letters can be used in any order?

 *r* selections = 7

*n* choices *=* 10 digits + 26 letters = 36 choices

*nr* = 367 = 78,364,164,096

There are more than 78 billion possibilities.

Example 2

How many six-character passwords can be made by combining lowercase letters, uppercase letters, numerals, and the characters @, $, &?

 *r* selections = 6

 *n* choices = 26 uppercases + 26 lowercases + 3 characters

 *nr* = 656 = 75,418,890,625

There are more than 75 billion possibilities.

**Permutations and Combinations**

Suppose that you coach a team of 4 runners. How many different ways you can put together a four-person relay team? You can choose any of the four runners for the first leg. Once you have chosen the first runner, you have 3 runners left to choose for the second leg. Then you have 2 runners left to choose for the third leg, and lastly, one person remains for the fourth leg. The total number of arrangements for the relay is 4x3x2x1 = 24. Each of the 24 different relay orders is called a **permutation**.

Whenever all selections come from a single group of items, no item may be selected more than once, and the order of arrangement matters. The total number of permutations possible with a group of *n* items is *n*!, where *n*! = n x (n-1) x …. x 2 x 1 is read “*n* factorial.”

Example 3

A high school administration needs to schedule 8 different classes – math, English, history, Spanish, science, gym, and 2 electives – in eight different time periods. How many different class schedules are possible?

 8! = 8x7x6x5x4x3x2x1 = 40320

Suppose you coach a team of 10 runners and can only select 4 runners for the relay. With 10 runners, you will have 10x9x8x7=5040 possibilities. However, by selecting only 4 runners, the can calculate by

$$\frac{10x9x8x7x6x5x4x3x2x1}{6x5x4x3x2x1}= \frac{10x9x8x7x6!}{6!}=\frac{10!}{\left(10-4\right)!}$$

If we make *r* selections from a group of *n* items, the number of permutations (order matters) is

*n*P*r* = $\frac{n!}{\left(n-r\right)!}$

If we make *r* selections from a group of *n* items, no item may be selected more than once, and the order of arrangement **does not** matter, the number of possible combinations is

*n*C*r* = $\frac{n!}{\left(n-r\right)!r!}$

Example 4

Suppose that you select 3 different flavors of ice cream in a shop that carries 12 flavors. How many flavor combinations are possible?

 *r* selections = 3

 *n* choices = 12

 *n*C*r* = $\frac{n!}{\left(n-r\right)!r!}$ = 12C*3* = $\frac{12!}{\left(12-3\right)!3!}$ = $\frac{12!}{9!3!}=\frac{12x11x10x9!}{9!3x}=\frac{12x11x10}{3x2x1}=\frac{1320}{6}=220$

Homework; #11-35 odds numbers only.