**Chapter 7: Unit B – Combining Probabilities**

Suppose you toss 2 dice and want to know the probability that both will come up 4.

 P(4) = $\frac{1}{6}$

 P(double 4) = $\frac{1}{6} $x $\frac{1}{6}$ = $\frac{1}{36}$ $\rightarrow $ joint probability

When the outcome of two events does not affect others, they are called **independent events**. Consider two independent events A and B with individual probability P(A) and P(B), the probability that A and B occur together is

 P(A and B) = P(A) x P(B)

This principle can be extended to any number of independent events.

 P(A and B and C) = P(A) x P(B) x P(C)

Example 1

Suppose you toss three coins. What is the probability of getting 3 tails?

 P(3 tails) = $\frac{1}{2}$ x $\frac{1}{2}$ x $\frac{1}{2}$ = $\frac{1}{8}$

Example 2

Suppose you toss 2 dice 3 times. What is the probability of getting two 3 followed a 5 on three tosses of a fair die?

 P(two 3, 5) = $\frac{1}{6}$ x $\frac{1}{6}$ x $\frac{1}{6}$ = $\frac{1}{216}$

Suppose that you pick a candy randomly from a box that initially contains 5 chocolates and 5 caramels. The probability of getting a chocolate/caramel on the first try is $\frac{5}{10}$ or $\frac{1}{2}$. Now suppose you pick a chocolate on the first try and eat it. What is the probability of getting chocolate on your second pick?

 P(chocolate1) = $\frac{5}{10}= \frac{1}{2}$

 P(chocolate2) = $\frac{4}{9}$

 P(chocolate x 2) = $\frac{1}{2}$ x $\frac{4}{9}$ = $\frac{4}{18}= \frac{2}{9}$

Two events are **dependent** if the outcome of one event affects the probability of the other event. The probability that dependent events A and B occur together is

 P(A and B) = P(A) x P(B given A)

This principle can be extended to any number of independent events.

 P(A and B) = P(A) x P(B given A) x P(C given A given B)

Example 3

A three-person jury must be selected at random from a pool of 12 people that has 6 men and 6 women. What is the probability of selecting an all male-jury?

 P(3 men) = $\frac{6}{12}$ x $\frac{5}{11}$ x $\frac{4}{10}$ = $\frac{120}{1320} ≈0.091$

Example 4

What is the probability of getting 2 red cards in a roll from a standard deck of cards?

 P(2 red cards) = $\frac{26}{52}$ x $\frac{25}{51}$ = $\frac{650}{2652} ≈ 0.25$

Two events are **non-overlapping**, if they cannot occur together. If A and B are non-overlapping events, the probability that either A or B occurs is

 P(A or B) = P(A) + P(B)

This principle can be extended to any number of independent events.

 P(A or B or C) = P(A) + P(B) + P(C)

Example 5

Suppose you roll a single die. What is the probability of rolling either a 2 or a 3?

 P(2 or 3) = $\frac{1}{6}+ \frac{1}{6}= \frac{2}{6}= \frac{1}{3}$

Example 6

Suppose you draw a card from a standard deck of cards. What is the probability of getting a queen or a heart?

P(queen or club) = $\frac{4}{52}+ \frac{13}{52}$ = $\frac{17}{52}$

In example 6, that is not the true probability because there is an overlapping event. Two events are **overlapping** if they can occur together. If A and B are overlapping events, then the probability that either A or B occurs is

 P(A or B) = P(A) + P(B) – P(A and B)

so, P(queen or club) = $\frac{4}{52}+ \frac{13}{52}- \frac{1}{52}$ = $\frac{16}{52}$

The *At Least Once* Rule

Suppose that you toss a coin four times. What is the probability of getting at least one head?

P(at least 1 head in 4 tosses) + P(no head in 4 tosses) = 1

P(at least 1 head in 4 tosses) = 1– P(no head in 4 tosses)

P(no head in 4 tosses) = P(no head) x P(no head) x P(no head) x P(no head)

 = [P(no head) x 4]

 = [P(no head)]4

P(at least 1 head in 4 tosses) = 1 – [P(no head)]4

 = 1 – ($\frac{1}{2}$)4

 = 1 $- \frac{1}{16}$ = $\frac{15}{16}$

If all trials are independent, the probability that event A occurs at least once in *n* trials is

 P(at least one event A in *n* trials) = 1 – P(no events A in *n* trials)

 = 1 – [P(not A in one trial)]n

Homework: 11-45, only ODDS