**Applications of College Algebra**

**Chapter 3 – Logic**

**Unit 3.5 – Equivalent Statements and Variations of Conditional**

**Equivalent Statements**

Equivalent compound statement s are made of simple statements and have the same corresponding truth values for all true-false combinations of these simple statements. If a compound statement is true, then its equivalent statement must also be true. Likewise, if a compound statement is false, its equivalent statement must also be false.

**Example 1 Show that Statements Are Equivalent**

1. Show that p ˅ ~q and ~p → ~q are equivalent.
2. Use the result from part (a) to write a statement that is equivalent to

The bill receives majority approval of the bill does not become law.

|  |  |  |  |
| --- | --- | --- | --- |
| p q | ~q | p ˅ ~q | ~p → ~q |
| T T | F | T | T |
| T F | T | T | T |
| F T | T | F | F |
| F F | T | T | T |

If the bill does not receive majority approval, then the bill does not become law.

The special symbol,$≡$, is used to show that two statements are equivalent.

 p ˅ ~q $≡$ ~p → ~q or ~p → ~q $≡$ p ˅ ~q

**Example 2 Show that Statements Are Equivalent**

Show that ~(~p) $≡$ p

|  |  |  |
| --- | --- | --- |
| p  | ~p | ~( ~p) |
| T  | F | T |
| F  | T | F |

The equivalent in example 2 illustrates that the double negative of a statement is equivalent to the statement.

Check Point: ~[~(~p)] $≡$ ~p

**Example 3 Equivalencies and Truth Tables**

d

c

a

Given

Select the statement that is not equivalent to

b

Miguel is blushing or sunburned.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| p q | p˅ q | ~q  | p→ ~ q | q ˅ p | ~p | ~p →q | ~q→p |
| T T | T | F | F | T | F | T | T |
| T F | T | T | T | T | F | T | T |
| F T | T | F | T | T | T | T | T |
| F F | F | T | T | F | T | F | F |

1. If Miguel is blushing, then he is not sunburned.
2. Miguel is sunburned or blushing.
3. If Miguel is not blushing, then he is sunburned.
4. If Miguel is not sunburned, then he is blushing.

**Variations of the Conditional Statement p → q**

Earlier section, we learned that p → q is equivalent to ~q → ~p. The truth value of a conditional statement does not change if the antecedent and the consequent are reverse then both of them are negated - **contrapositive**.

**A Conditional Statement and Its Equivalent Contrapositive**

p → q $≡$ ~q → ~p

|  |
| --- |
| **Variations of the Conditional Statement** |
| Name | Symbolic Form | English Translation |
| Conditional | p→q | If p then q |
| Converse | q→p | If q then p |
| Inverse | ~p→~q | If not p then not q |
| Contrapositive | ~q→~p | If not q then not p |

Conditional and converse are not equivalent.

Conditional and inverse are not equivalent.

Conditional and contrapositive are equivalent.

Converse and inverse are equivalent.

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