**Application of College Algebra**

**Chapter 3: Logic**

**Unit 3.4: Truth Table for Conditional and Biconditional**

**Conditional Statements, →**

An example from previous section:

p: The person is a father.

q: The person is a male.

Case 1 (T, T) If the person is a father, then the person is a male. T

Case 2 (T, F) If the person is a father, then the person is not a male. F

Case 3 (F, T) If the person is not a father, then the person is a male. T

Case 4 (F, F) If the person is not a father, then the person is not a male. T

**The Definition of Conditional**

A conditional is false only when the antecedent is true and the consequent is false.

|  |  |
| --- | --- |
| p q | p → q |
| T T | T |
| T F | F |
| F T | T |
| F F | T |

**Example 1 Constructing a Truth Table**

Construct a truth table for ~q → ~p

|  |  |  |  |
| --- | --- | --- | --- |
| p q | ~q | ~p | ~q → ~p |
| T T | F | F | T |
| T F | T | F | F |
| F T | F | T | T |
| F F | T | T | T |

Check Point

Construct a truth table for ~p → ~q

|  |  |  |  |
| --- | --- | --- | --- |
| p q | ~p | ~q | ~p → ~q |
| T T |  |  |  |
| T F |  |  |  |
| F T |  |  |  |
| F F |  |  |  |

**Example 2 Constructing a Truth Table**

Construct a truth table for [(p ∨ q) ∧ ~p] → q

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| p q | p ∨ q | ~p | (p ∨ q) ∧ ~p | [(p ∨ q) ∧ ~p] → q |
| T T | T | F | F | T |
| T F | T | F | F | T |
| F T | T | T | T | T |
| F F | F | T | F | T |

Some compound statements are false in all possible cases. Such statements are called self-contradictions. An example of a self-contradiction is the statement p ∧ ~p

|  |  |  |
| --- | --- | --- |
| p | ~p | p ∧ ~p |
| T | F | F |
| F | T | F |

If p represents “I am going,” the p ∧ ~p translate as “I am going and I am not going.”

**Example 3 Constructing a Truth Table with Eight Cases**

“…If the public concludes that the IRS cannot meet these basic expectations, the risk to the tax system will become very high and the effects very difficult to reverse (The New York Time, February 13, 2000)”

p: The public concludes that eh IRS can meet basic expectations.

q: The risk to the tax system will become very high.

r: The effects will be very difficult to reserve.

~p → (q ∧ r)

|  |  |  |  |
| --- | --- | --- | --- |
| p q r | ~p | q ∧ r | ~p → (q ∧ r) |
| T T T | F | T | F |
| T T F | F | F | T |
| T F T | F | F | T |
| T F F | F | F | T |
| F T T | T | T | T |
| F T F | T | F | F |
| F F T | T | F | F |
| F F F | T | F | F |

In section 3.2, we learned the connective biconditional (↔), translated “if and only if” respectively. The biconditional statement p ↔ q means that

p → q and q → p. We write this symbolically as (p → q) ∧ (q → p).

To create a truth table for p ↔ q, we will first make a truth table for the conjunction of the two conditionals p → q and q → p.

The conditional is false only when p is true and q is false.

|  |  |  |  |
| --- | --- | --- | --- |
| p q | p → q | q → p | (p → q) ∧ (q → p) |
| T T | T | T | T |
| T F | F | T | F |
| F T | T | F | F |
| F F | T | T | T |

The conditional is false only when q is true and p is false.

An example from previous section,

p: A person is unmarried male.

q: A person is a bachelor.

**The Definition of the Biconditional**

A biconditional is true only when the component statements have the same value.

|  |  |
| --- | --- |
| p q | p ↔ q |
| T T | T |
| T F | F |
| F T | F |
| F F | T |

**The Definitions of Symbolic Logic**

1. Negation, ~: not

The negation of a statement has the opposite meaning, as well as the opposite truth value, from the statement.

1. Conjunction, ∧: and

The only case in which a conjunction is true is when both component statements are true.

1. Disjunction, ∨: or

The only case in which a disjunction is false is when both component statements are false.

1. Conditional, →: if-then

The only case in which a conditional is false is when the first component statement the antecedent, is true and the second component statement the consequence is false.

1. Biconditional, ↔: if and only if, iff

A biconditional is true only when the component statements have the same truth value.

Example 4 Constructing a Truth Table

Construct a truth table for (p ∨ q) ↔ (~q → p)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| p q | p ∨ q | ~q | ~q → p | (p ∨ q) ↔ (~q → p) |
| T T | T | F | T | T |
| T F | T | T | T | T |
| F T | T | F | T | T |
| F F | F | T | F | T |

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