**Applications of College Algebra**

**Chapter 3: Logic**

**Unit 3.3: Truth Tables for Negation, Conjunction, and Disjunction**

**Negation, ~**

The negation of a true statement is a false statement. We can express this in a table in which T represents true and F represents false.

|  |  |
| --- | --- |
| p | ~p |
| T | F |

The negation of a false statement is a true statement. This, too, can be shown in the table form.

|  |  |
| --- | --- |
| p | ~p |
| F | T |

Combining the two tables is called the truth table for negation. This truth table expresses the idea that ~p has the opposite truth value from p.

|  |
| --- |
| **Negation** |
| p | ~p |
| T | F |
| F | T |

**Conjunction, ˄**

A friend tells you, “I visited London and I visited Paris.” In order to understand the truth values for this statement, we break it down into its two simple statements:

p: I visited London.

q: I visited Paris.

There are four possible cases to consider.

Case 1. You friend actually visited both cities.

|  |  |
| --- | --- |
| p q | p ˄ q |
| T T | T |

Case 2. Your friend only visited London.

|  |  |
| --- | --- |
| p q | p ˄ q |
| T F | F |

Case 3. Your friend only visited Paris.

|  |  |
| --- | --- |
| p q | p ˄ q |
| F T | F |

Case 4. Your friend did not visit either city.

|  |  |
| --- | --- |
| p q | p ˄ q |
| F F | F |

A conjunction is true only when both simple statements are true.

|  |
| --- |
| **Conjunction** |
| p q | p ˄ q |
| T T | T |
| T F | F |
| F T | F |
| F F | F |

|  |
| --- |
| **Statements of Conjunction and Their Truth Values** |
| **Statement** | **Truth Value** | **Reason** |
| 3+2=5 and London is in England | T | Both simple statements are true. |
| 3+2=5 and London is in France | F | Second simple statement is false. |
| 3+2=6 and London is in England | F | First simple statement is false. |
| 3+2=6 and London is in France | F | Both simple statements are false. |

\*\*\* If one simple statement is false, then the conjunction is false.

The statements that come before and after the main connective in a compound statement do not have to be simple statement.

 (~p ˅ q) ˄ ~ q

The statements that make up this conjunction are ~p ˅ q and ~q. The conjunction is true only when both ~p ˅ q and ~q are true. Noticed that ~p ˅ q is not a simple statement. We call ~p ˅ q and ~q the ***component statements*** of the conjunction.

**Disjunction, ˅**

Now your friend states, “I will visit London or I will visit Paris.” We presume that this is the inclusive “or,” if your friend visits either or both of these cities, the true has been told.

The disjunction is false only in the event that neither city is visited, which mean both component statements are false.

|  |
| --- |
| **Disjunction** |
| p q | p ˅ q |
| T T | T |
| T F | T |
| F T | T |
| F F | F |

|  |
| --- |
| **Statements of Disjunction and Their Truth Values** |
| **Statement** | **Truth Value** | **Reason** |
| 3+2=5 or London is in England | T | Both component statements are true. |
| 3+2=5 or London is in France | T | First component statement is true. |
| 3+2=6 or London is in England | T | Second component statement is true. |
| 3+2=6 or London is in France | F | Both component statements are false. |

\*\*\*If one component statement is true, then the disjunction is true.

**Example 1 Using Definitions of Negation, Conjunction, and Disjunction**

Let p and q represent the following statements:

 p: 10 $>$ 4

 q: 3 $<$ 5

Determine the truth value for each statement:

1. p ˄ q b. ~p ˄ q
2. p ˅ q d. ~p ˅ ~q

Check Point

Let p and q represent the following statements:

 p: 3 + 5 = 8

 q: 2x7 = 20

Determine the truth value for each statement:

1. p ˄ q b. p ˄ ~q

c. ~p ˅ q d. ~p ˅ ~q

**Constructing Truth Tables**

The truth tables can be used to gain a better understanding of English statements. The truth tables are based on the definitions of negation, conjunction, and disjunction.

When constructing truth tables for compound statements containing only the simple statements p and q.

1. List the four possible combinations of truth values for p and q

|  |  |
| --- | --- |
| p q |  |
| T T |  |
| T F |  |
| F T |  |
| F F |  |

1. Determine the column heading by reconstructing the compound statement one component statement at a time. The final column heading should be given compound statement.

1. Use each column heading to fill in the four truth values: ~, ˄, ˅

**Example 2 Construct a Truth Table**

Construct a truth table for ~(p ˄ q)

|  |  |
| --- | --- |
| p q1 |  |
| T T |  |
| T F |  |
| F T |  |
| F F |  |

|  |  |
| --- | --- |
| p q2 | p ˄ q |
| T T | T |
| T F | F |
| F T | F |
| F F | F |

|  |  |  |
| --- | --- | --- |
| p q3 | p ˄ q | ~(p ˄ q) |
| T T | T | F |
| T F | F | T |
| F T | T | F |
| F F | F | T |

Negating

The final column in the truth table for ~(p ˄ q) tells us that the statement is false only when both p and q are true. For example,

 p: Harvard is a college (true).

 q: Yale is a college (true).

The statement ~(p ˄ q) translate as

 It is not true that Harvard and Yale are colleges.

This compound statement is false. It is true that Harvard and Yale are colleges.

Check Point

Construct a truth table for ~(p ˅ q) to determine when the statement is true and when the statement is false.

|  |  |
| --- | --- |
| p q1 |  |
| T T |  |
| T F |  |
| F T |  |
| F F |  |

|  |  |  |
| --- | --- | --- |
| p q | p ˅ q | ~(p ˅ q) |
| T T |  |  |
| T F |  |  |
| F T |  |  |
| F F |  |  |

|  |  |
| --- | --- |
| p q2 | p q3 |
| T T |  |
| T F |  |
| F T |  |
| F F |  |

Example 3 Constructing a Truth Table

Construct a truth table for ~p ˅ ~ q

|  |  |  |  |
| --- | --- | --- | --- |
| p q | ~p | ~q | ~p ˅ ~q |
| T T | F | F | F |
| T F | F | T | T |
| F T | T | F | T |
| F F | T | T | T |

Check Point

Construct a truth table for ~p ˄ ~q

|  |  |  |  |
| --- | --- | --- | --- |
| p q | ~p | ~q | ~p ˄ ~q |
| T T |  |  |  |
| T F |  |  |  |
| F T |  |  |  |
| F F |  |  |  |

Example 4 Constructing a Truth Table

Construct a truth table for (~p ˅ q) ˄ ~q

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| p q | ~p | ~p ˅ q | ~ q | (~p ˅ q) ˄ ~q |
| T T | T | T | F | F |
| T F | F | F | T | F |
| F T | T | T | F | F |
| F F | T | T | T | T |

Check Point

Construct a truth table for (p ˄ ~q) ˅ ~p

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| p q | ~q | p ˄ ~q | ~ p | (p ˄ ~q) ˅ ~p |
| T T |  |  |  |  |
| T F |  |  |  |  |
| F T |  |  |  |  |
| F F |  |  |  |  |

\*\*\* A compound statement that is always true is called a **tautology**.

**Example 5** Construct a truth table for p ˅ ~p

|  |  |  |
| --- | --- | --- |
| p | ~p | p ˅ ~p |
| T | F | T |
| F | T | T |

There are several types of tautology which are commonly used in everyday life, in poetry, in prose, in [songs](http://literarydevices.net/tag/songs/), and in discussions depending on the requirements of a situation. Some of the common categories are:

* Due to inadequacies in Language
* Intentional ambiguities
* Derision

“To be or not to be…”

‘‘

* As a Poetic Device
* Psychological significance
* Used by inept Speakers

Some compound statements involve three simple statements, usually represented by p, q, and r. In this situation, there are eight different true-false combinations.

|  |  |  |
| --- | --- | --- |
| p | q | r |
| T | T | T |
| T | T | F |
| T | F | T |
| T | F | F |
| F | T | T |
| F | T | F |
| F | F | T |
| F | F | F |

**Example 6 Constructing a Truth Table with Eight Cases**

1. Construct a truth table for the following statement:

I study hard and ace the final, or I fail the course.

1. Suppose that you study hard, you do not ace the final, you fail the course. Under these conditions, is the compound statement is part (a) true or false?

First, assign letters for each simple statement.

 p: I study hard.

 q: I ace the final.

 r. I fail the course

1. (p ˄ q) ˅ r

|  |  |  |
| --- | --- | --- |
| p q r | p ˄ q | (p ˄ q) ˅ r |
| T T T | T | T |
| T T F | T | T |
| T F T | F | T |
| T F F | F | F |
| F T T | F | T |
| F T F | F | F |
| F F T | F | T |
| F F F | F | F |

1. Under case 3, it would be true.

**Homework: 1-31, 43-47, 53-61 odd**