**Applications of College Algebra**

**Chapter 3**

**Unit 3.2 – Compound Statements and Connectives**

Statements formed by combining two or more simple statements. Words are used to join these simple statements are called connectives. Connectives include words such as **and, or, if … then**, and **if and only if.**

***And* Statements**

If *p* and *q* represent two simple statements, then **the compound statement “*p* and *q*” is symbolized by *p* ∧ *q*.** The compound statement formed by connecting statements with the word and is called a conjunction. The symbol for *and* is ∧.

**Example 1 Translating from English to symbolic Form**

Let p and q represent the following simple statements:

*p*: It is after 5 P.M.

*q*: They are working.

Write each compound statement below in the symbolic form:

1. It is after 5 P.M. and they are working.
2. It is after 5 P.M. and they are not working.
3. They are working and it is after 5 P.M.
4. It is not after 5 P.M. and they are working.

Let *p* and *q* represent the following simple statements:

*p*: Students prepared for assessment.

*q*: Students received good grade.

Write each compound statement below in symbolic form:

1. Student did not prepare for assessment and students did not receive good grade.
2. Students did not received good grade and students did not prepare for assessment.

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| **Common English Expression for *p*∧ *q*** | | |
| **Symbolic Statement** | **English Statement** | **Example**  *p*: It is after 5 pm.  *q*: They are working. |
| *p* ∧ *q* | *p* and *q* | It is after 5 P.M. and they are working. |
| *p* ∧ *q* | *p* but *q* | It is after 5 P.M., but they are working. |
| *p* ∧ *q* | *p* yet *q* | It is after 5 P.M., yet they are working. |
| *p* ∧ *q* | *p* nevertheless *q* | It is after 5 P.M.; nevertheless, they are working. |

Not every English statement with the word *and* is a conjunction.

Not a conjunction: “Nonviolence and truth are inseparable.” Ghandi.

Conjunction: Pizza and beer are not recommended for people with ulcer.

Conjunction combines two independent statements.

**Or Statements**

The connective *or* can mean two different things: exclusive and inclusive.

I visited London or Paris

*Exclusive* *or*: I visited London or Paris, but not both.

*Inclusive or*: I visited London or Paris or both.

In this chapter and in mathematics in general, when the connective *or* appears, it means the *inclusive or*. The compound statement formed by connecting statements with the word *or* is called a **disjunction. The symbol for or is ∨. Thus, we can write symbolize the compound statement “*p* or *q* or both” by *p* ∨ *q*.**

**Example 2 Translating from English to Symbolic Form**

Let *p* and *q* represent the following simple statements:

*p:* The bill receives majority approval.

*q:* The bill becomes a law.

Write each compound statement below in symbolic form:

1. The bill receives majority approval or the bill becomes a law. ***p* ∨ *q*.**
2. The bill receives majority approval or does not become a law. ***p* ∨ ~ *q.***

Let *p* and *q* represent the following simple statements:

*p*: You graduate.

*q*: You satisfy the math requirement.

Write each compound statement below in symbolic form:

1. You graduate or you satisfy the math requirement.
2. You satisfy the math requirement or you do not graduate.

If-Then Statements

The compound statement formed by connecting statements with “if-then” is called a **conditional statement**. The symbol for “if-then” is →. In a conditional statement, the statement before the → connective is called the **antecedent**. The statement after the → is called the **consequent**.

**Example 3 Translating from English to Symbolic Form**

Let *p* and *q* represent the following simple statements:

*p:* A person is a father.

*q:* A person is a male.

Write each compound statement below in symbolic form:

1. If a person is a father, then that person is a male.
2. If a person is a male, then that person is a father.
3. If a person is not a male, then that person is not a father.
4. If a person is not a father, then that person is not a male.
5. If a person is a male, then that person is not a father.

Conditional statements in English often omit the word *then* and simply use a comma. When then is included, the comma can be included or omitted.

If a person is a father, then that person is a male.

If a person is a father then that person is a male.

If a person is a father, that person is a male.

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| **Common English Expressions for *p* → *q*** | | |
| **Symbolic Statement** | **English Statement** | **Example**  ***p*: A person is a father.**  ***q:* A person is a male.** |
| *p* → *q* | If *p* then *q.* | If a person is a father, then that person is a male. |
| *p* → *q* | *q* if *p*. | A person is a male if that person is a father. |
| *p* → *q* | *p* is sufficient for *q*. | Being a father is sufficient for being a male. |
| *p* → *q* | *q* is necessary for *p*. | Being a male is necessary for being a father. |
| *p* → *q* | *p* only if *q*. | A person is a father only if that person is a male. |
| *p* → *q* | Only if *q, p*. | Only if a person is a male is that person a father. |

**Example 4 Translating from English to Symbolic Form**

Let *p* and *q* represent the following simple statements:

*p:* We suffer huge budget deficits.

*q:* We control military spending.

Write the following statement in symbolic form:

Controlling military spending is necessary for not suffering huge budget deficits. **~ *p* → *q*.**

Suffering huge budget deficits is necessary for not controlling military spending. **~ *q* → *p.***

***If and Only If* Statements**

In a conditional statement is true, reversing the antecedent and consequent may result in a statement that is not necessarily true

If a person is a father, then that person is a male.

If a person is a male, then that person is father.

However, some true conditional statements are still true when the antecedent and consequent are reversed.

If a person is an unmarried male, then that person is a bachelor.

If a person is a bachelor, then that person is an unmarried male.

Rather than deal with two separate conditionals, we can combine them into one *biconditional statement*.

If a person is an unmarried male if and only if that person is a bachelor.

If *p* and *q* represent two simple statements, then **the compound statement “*p* if and only if *q”* is symbolized by *p* ↔ *q****.*

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| **Words with the Most Meaning in the *Oxford English Dictionary*** | |
| Word | Meanings |
| Set | 464 |
| Run | 396 |
| Go | 368 |
| Take | 343 |
| Stand | 334 |

**Example 5 Translating from English to Symbolic Form**

*p:* The word is *set.*

*q:* The word has 464 meanings.

Write each of the compound statements below in its symbolic form:

1. The word is *set* if and only if the word has 464 meanings. ***p* ↔ *q.***
2. The word does not have 464 meanings if and only if the word is not *set.* **~ *q ↔* ~ *p*.**

Let *p* and *q* represent the following statements:

*p:* The word is *run.*

*q:* The word has 396 meanings.

Write each of the compound statements below in its symbolic form:

1. The word has 396 meanings if and only if the word is *run.*
2. The word is not *run* if and only if the word does not have 396 meanings.

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| **Statement of Symbolic Logic** | | |
| **Name** | **Symbolic Form** | **Common English Translations** |
| **Negation** | ~*p* | Not *p.* It is not true that *p.* |
| **Conjunction** | *p* ∧ *q* | *p* and *q. p* but *q.* |
| **Disjunction** | *p* ∨ *q* | *p* or *q.* |
| **Conditional** | *p → q* | If *p,* then *q.* If *p*  is sufficient for *q. q* is necessary for *p.* |
| **Biconditional** | *p* ↔ *q* | *p* if and only if *q. p* is necessary and sufficient for *q.* |

**Symbolic Statements with Parentheses**

Parentheses in symbolic statements indicate which statements are to be grouped together. For example, ~ (*p* ∧ *q*) means the negation of the entire statement *p* and *q.* By contrast, ~ *p* ∧ *q* means that only *p* is negated.

Example 6 Expressing Symbolic Statements with and without Parentheses

Let *p* and *q* represent the following statements:

*p:* She is wealthy.

*q:* She is happy.

Write each of the following statements in words:

1. ~(*p* ∧ *q*)
2. ~ *p* ∧ *q*
3. ~(*p* ∨ *q*)
4. ~*q* ∧ *p*
5. *q* ∨ ~*p*

Let *p* and *q* represent the following simple statements:

*p*: He earns $105,000 yearly.

*q*: He is often happy.

Write each of the following symbolic statements in words:

1. ~ (*p*∧ *q*)
2. ~ *q* ∧ *p*
3. ~(*q* → *p*)

Many compound statements contain more than one connective. When expressed symbolically, parentheses are used to indicate which simple statements are group together.

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| **Symbolic Statement** | **Statement to Group Together** | **English Translation** |
| (*q* ∧ ~ *p*) → ~  *r* | *q* ∧ ~ *p* | If *q* and not *p*, then not *r*. |
| *q* ∧ (~ *p* →~ *r*) | ~ *p* → ~ *r* | *q*, and if not *p* then not *r*. |

Example 7 Expressing Symbolic Statements with Parentheses in English

Let *p, q,* and *r* represent the following simple statements:

*p:* A student misses lecture.

*q:* A student studies.

*r:* A student fails.

Write each of the following symbolic statements below in words:

1. (*q* ∧ ~ *p*) → ~ *r*
2. *q* ∧ (~ *p* → ~ *r*)

Let *p, q,* and *r* represent the following simple statements:

*p:* The plant is fertilized.

*q:* The plant is not watered.

*r:* The plant wilts.

Write each of the following symbolic statements below in words:

1. (*p* ∧ ~ *q*) → ~ *r*
2. *p* ∧ (~ *q* → ~ *r*)
3. ~ *r* → (*p* ∧ ~ *q*)

**Dominance of Connectives**

If a symbolic statement appears without parentheses, statements before and after the most dominant connective should be grouped. Symbolic connectives are categorized from the least dominant, negation, to the most dominant, the biconditional.

The dominance of connectives used in symbolic logic is defined in the following order:

1. Negation, ~ 2. Conjunction, ˄ 3. Conditional, → 4. Biconditional, ↔

Disjunction, ˅

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| **Using Dominance of Connectives** | | | |
| **Statement** | **Most Dominance Connective** | **Statement’s Meaning Clarified with Grouping Symbols** | **Type of Statement** |
| *p* → *q* ˄ ~ *r* | *p* → *q* ˄ ~ *r* | *p* → (*q* ˄ ~ *r*) | Conditional |
| *p* ˄ *q* → ~ *r* | *p* ˄ *q* → ~ *r* | (*p* ˄ *q*)→ ~ *r* | Conditional |
| *p* ↔ *q* → *r* | *p* ↔ *q* → *r* | *p* ↔ (*q* → *r*) | Biconditional |
| *p* → *q*  ↔ *r* | *p* → *q*  ↔ *r* | (*p* → *q*) ↔ *r* | Biconditional |
| *p* ˄~ *q* → *r* ˅ *s* | *p* ˄~ *q* → *r* ˅ *s* | (*p* ˄~ *q*)→ (*r* ˅ *s*) | Conditional |
| *p* ˄ *q* ˅ *r* | ˄ and ˅ have the same level of dominance. | The meaning is ambiguous. | ? |

**Example 8 Using the Dominance of Connectives**

Write each compound statement below in symbolic form:

1. I do not fail this course if and only if I study hard and pass the final.
2. I do not fail this course if and only if I study hard, and pass the final.

First, assign a letter to represent each simple statement that is not negated.

*p:* I fail this course.

*q:* I study hard.

*r:* I pass the final.

1. **~ *p* ↔ (*q* ˄ *r*).**
2. **(~ *p* ↔ *q*) ˄ *r.***

Write each compound statement below in symbolic form:

1. If there is too much homework or a teacher is boring then I do not take that class.
2. There is too much homework, or if a teacher is boring then I do not take that class.

**Homework: Exercise Set 3.2 #1-65 odd and 67-99 odd.**

**Thursday 3/10/2016: Quiz on 3.1-3.2 and homework are due.**