**Applications of College Algebra**

**Chapter 3 Logic**

**Unit 3.1 – Statements, Negations, and Quantified Statements**

**Statements and Using symbols to Represent Statements**

A **statement** is a sentence that is either true or false, but not both simultaneously.

1. London is the capital of England.
2. William Shakespeare wrote the last episode of *The Sopranos*.
3. Read pages 23-27.
4. If I start losing my memory, how will I know?
5. *Titanic* is the greatest movies of all time.

In symbolic logic, we use lowercase letters such as *p, q, r* and *s* to represent statements.

*p*: London is the capital of England.

*q*: William Shakespeare wrote the last episode of *The Sopranos*.

**Negating Statements**

The sentence “London is the capital of England” is a true statement. The *negation* of this statement is “London is not the capital of England,” is a false statement. The **negation** of a true statement is false statement and the negation of a false statement is a true statement.

**Example 1 Forming Negation**

1. Shakespeare wrote the last episode of *The Sopranos.*
2. Today is not Monday.
3. Arizona is the 48th state.
4. Late start does not occur every Wednesday at Page High School.

Form the negation of each statement:

1. Paris is the capital of Spain.
2. July is not a month.

The negation of statement *p* is expressed by writing ~*p*. We read this as “not *p*” or “It is not true that *p*.”

**Example 2 Expressing Negations Symbolically**

Let *p* and *q* represent the following statements:

*p:* Shakespeare wrote the last episode of *The Sopranos.*

*q:* Today is not Monday.

Express each of the following statements symbolically:

1. Shakespeare did not write the last episode of *The Sopranos.* ***~p.***
2. Today is Monday.***~q.***

**Example 3 Translating a Symbolic Statement into Words**

Let *p* represent the following statement:

*p*: The United States has the world’s highest divorce rate.

Express the symbolic statement *~p* in words.

The United States does not have the world’s highest divorce rate.

Let *q* represent the following statement:

*q*: It is not cold in Page in the winter.

Express the symbolic statement *~q* in words.

Let *r* represent the following statement:

*r*: Chicago O’Hare is the world’s business airport.

Express the symbolic statement *~r* in words.

**Quantified Statements**

Statements contain the following words **all, some**, and **no** (or **none**). These words are called **quantifiers**.

All poets are writes

Some people are bigots.

No common colds are fatal.

Some students do not work hard.

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| **Equivalent ways of Expressing Quantified Statements** | | |
| **Statement** | **Equivalent Way** | **Example** |
| All A are B. | There are no A that are not B | All poets are writers.  There are not poets that are not writers. |
| Some A are B. | There exists at least one A that is a B. | Some people are bigots.  There at least one person is a bigot. |
| Not A are B. | All A are not B. | No common colds are fatal.  All common colds are not fatal. |
| Some A are not B. | Not all A are B. | Some students do not work hard.  Not all students work hard. |

Forming the negation of a quantified statement can be a bit tricky. Suppose we want to negate the statement “All writers are poets.” Because this statement is false, its negative must be true. The negation is “Not all writers are poets.” This means the same thing as “Some writers are poets.”

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| **Negations of Quantified Statements** | | |
| **Statement** | **Negation** | **Example** |
| All B are B. | Some A are not B | All people take exams honestly.  Negations:  Some people do not take exams honestly. |
| Some A are B. | Not A are B. | Some roads are open.  Negation: No roads are open. |

**Study Tip**

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| The statements diagonally opposite each other are negations.  All A are B. No A are B.  Some A are B. Some A are not B. |

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| **Examples Negations of Quantified Statements** | |
| **Statement** | **Negation** |
| All human are mortal. | Some human are not mortal. |
| Some students do not come to class prepared. | All students come to class prepared. |
| Some psychotherapists are in therapy. | No psychotherapists are in therapy. |
| No well-behaved dogs shred couches. | Some well-behaved dogs shred couches. |

**Example 4 Negating Quantified Statement**

All piston rings were replaced.

Negative:

Some piston rings were not replaced.

At least one piston rings was not replaced.

**Homework: Exercise Set 3.1 #15-56 odd**