9. One person in a stadium filled with 100,000 people is chosen at random to win a free pair of airline tickets. What is the probability that it will *not* be you?

   a. 1 in 100,000 b. 0.99 c. 0.99999

10. There are 365 possible birthdays in a year. In a class of 25 students, the chance of finding 2 students with the same birthday is

    a. 25/365. b. $2 \times 25/365$. c. greater than 0.5.

Exercises

REVIEW QUESTIONS

1. What are arrangements with repetition? Give an example of a situation in which the n^r formula gives the number of possible arrangements.
2. What do we mean by *permutations*? Explain the meaning of each of the terms in the permutations formula. Give an example of its use.
3. What do we mean by *combinations*? Explain the meaning of each of the terms in the combinations formula. Give an example of its use.
4. Explain what we mean when we say that *some* outcome is much more likely than a particular outcome. How does this idea affect our perception of coincidences?

DOES IT MAKE SENSE?

Decide whether each of the following statements makes sense (or is clearly true) or does not make sense (or is clearly false). Explain your reasoning.

5. I used the permutations formula to determine how many possible relay orders we could make with the 10 girls on our swim team.
6. I used the combinations formula to determine how many different five-card poker hands are possible.
7. The number of different possible batting orders for 9 players on a 25-person baseball team is so large that there's no hope of trying them all out.
8. It must be my lucky day, because the five-card poker hand I got had only about a 1 in 2.5 million chance of being dealt to me.
9. The probability that two people in a randomly selected group will have the same last name is much higher than the probability that someone will have the same last name as I do.
10. Someone wins the lottery every week, so I figure that if I keep playing eventually I will be the one who wins.

BASIC SKILLS & CONCEPTS

11–22: Review of Factorials. Use the skills covered in the Brief Review on p. 459 to evaluate the following quantities *without* using the factorial key on your calculator (you may use the multiplication key). Show your work.

11. $6!$ 12. $12!$ 13. $\frac{5!}{3!}$

14. $\frac{10!}{8!}$ 15. $\frac{12!}{4!\,3!}$ 16. $\frac{9!}{4!\,2!}$

17. $\frac{11!}{3!(11-3)!}$ 18. $\frac{30!}{29!}$ 19. $\frac{8!}{3!(8-3)!}$

20. $\frac{30!}{28!}$ 21. $\frac{6!\,8!}{4!\,5!}$ 22. $\frac{15!}{2!\,13!}$

23–40: Counting Methods. Answer the following questions using the appropriate counting technique, which may be either arrangements with repetition, permutations, or combinations. Be sure to explain why this counting technique applies to the problem.

23. How many different nine-digit zip codes can be formed?
24. How many different five-character passwords can be formed from the lowercase letters of the alphabet?
25. How many different five-character passwords can be formed from the lowercase letters of the alphabet if repetition is not allowed?
26. A city council with nine members must elect a four-person executive committee consisting of a mayor, deputy mayor, secretary, and treasurer. How many executive committees are possible?
27. How many ways can the nine performances at a piano recital be ordered?
28. A city council with nine members must appoint a three-person subcommittee. How many subcommittees are possible?
29. Suppose you have 20 CDs from which you choose 6 CDs to put in the CD player in your car. If you are not particular about the order, how many 6-CD sets are possible?
30. How many 6-person lineups can be formed from a 15-player volleyball roster, assuming every player can be assigned to any position?
31. How many different birth orders with respect to gender are possible in a family with five children? (For example, BBBGG and BGBGG are different orders.)
32. How many different 5-card hands can be dealt from a 52-card deck?
33. How many license plates can be made of the form *XX–YYYY*, where *X* is a letter of the alphabet and *Y* is a numeral 0–9?
34. How many different groups of six balls can be drawn from a barrel containing balls numbered 1–36?
35. How many different telephone numbers of the form *aaa-bbb-cccc* can be formed if the area code *aaa* cannot contain 0 and the prefix *bbb* cannot contain 9?
36. How many anagrams (rearrangements) of the letters ILOVEMATH can you make?

37. How many different three-letter "words" can be formed from the genetic alphabet ACGT?

38. The debate club has 15 members, but only 5 can compete at the next meet. How many 5-person teams are possible?

39. A recording engineer wants to make a CD with 12 songs. In how many different ways can the CD be made?

40. A dog shelter is giving away 15 different dogs, but you have room for only 4 of them. How many different dog families could you have?

41–42: **Birthday Coincidences.** Suppose you are part of a group of people at a dinner party. For the situations given, find the probability that at least one of the other guests has *your* birthday and the probability that *some* pair of guests shares the same birthday. Discuss your results. (Assume 365 days in a year.)

41. You are one of 12 people at the party.

42. You are one of 20 people at the party.

FURTHER APPLICATIONS

43. **Ice Cream Shop.** Josh and John's Ice Cream Shop offers 20 different flavors of ice cream and 8 different toppings. Answer the following questions by using the appropriate counting technique (multiplication principle, arrangements with repetitions, permutations, or combinations). Explain why you chose the particular counting technique.

a. How many different sundaes can you create using one of the ice cream flavors and one of the toppings?

b. How many different triple cones can you create from the 20 flavors if the same flavor may be used more than once? Assume that you specify which flavor goes on the bottom, middle, and top.

c. Using the 20 flavors, how many different triple cones can you create with 3 *different* flavors if you specify which flavor goes on the bottom, middle, and top?

d. Using the 20 flavors, how many different triple cones can you create with 3 *different* flavors if you don't care about the order of the flavors on the cone?

44. **Telephone Numbers.** A ten-digit phone number in the United States consists of a three-digit area code followed by a three-digit *exchange* followed by a four-digit number.

a. The first digit of the area code cannot be 0 or 1. The first digit of the exchange cannot be 0 or 1. How many different ten-digit phone numbers can be formed? Can a city with 2 million telephone numbers be served by a single area code? Explain.

b. How many exchanges are needed to serve a city of 80,000 people? Explain.

45. **Pizza Hype.** Luigi's Pizza Parlor advertises 56 different three-topping pizzas. How many individual toppings does Luigi actually use? Ramona's Pizzeria advertises 36 different two-topping pizzas. How many toppings does Ramona actually use? (*Hint:* In these problems, you are given the total number of combinations, and you must find the number of toppings that are used.)

46. **ZIP Codes.** The U.S. Postal Service uses both five-digit and nine-digit ZIP codes.

a. How many five-digit ZIP codes are available to the U.S. Postal Service?

b. For a U.S. population of 300 million people, what is the average number of people per five-digit ZIP code if all possible ZIP codes are used? Explain.

c. How many nine-digit ZIP codes are available to the U.S. Postal Service? Could everyone in the United States have his or her own personal nine-digit ZIP code? Explain.

47–54: **Counting and Probability.** Find the probability of the given event.

47. Choosing six numbers that match six randomly selected balls when the balls are numbered 1 through 36

48. Choosing five numbers that match five randomly selected balls when the balls are numbered 1 through 42

49. Being dealt 5 cards from a standard 52-card deck, and the cards are a 10, jack, queen, king, and ace, all of the same suit

50. Guessing the top three winners (in order) from a group of ten finalists in a soccer tournament

51. Guessing the top three winners (in any order) from a group of 15 finalists in a spelling bee

52. Randomly selecting 3 Ohio students from a group of 16 students, 7 of whom are from Ohio

53. Being dealt 5 cards from a standard 52-card deck and getting four of a kind (for example, four aces)

54. Being in the first half of the program when you are one of ten performers whose order of performance is randomly selected

55. **Hot Streaks.** Suppose that 2000 people are all playing a game for which the chance of winning is 48%.

a. Assuming everyone plays exactly five games, what is the probability of one person winning five games in a row? On average, how many of the 2000 people could be expected to have a "hot streak" of five games?

b. Assuming everyone plays exactly ten games, what is the probability of one person winning ten games in a row? On average, how many of the 2000 people could be expected to have a "hot streak" of ten games?