BY THE WAY

By comparing IQ tests and questions that have stayed the same over the years, Dr. James Flynn discovered a startling fact: Raw scores on IQ tests (before they are set to a mean of 100) rose steadily during the 20th century, so that someone scoring an "intellectually deficient" IQ of 70 today would have rated an "intellectually superior" IQ of 130 a century ago. It seems unlikely that people today are really so much smarter than their parents and grandparents, but no one has yet found a clear explanation for Flynn's discovery.

Standard Scores and Percentiles

Once we know the standard score of a data value, the properties of the normal distribution allow us to find its percentile in the distribution. You are probably familiar with the idea of percentiles. For example, if you scored in the 45th percentile on the SAT, 45% of the SAT scores were lower than yours.

PERCENTILES

The *n*th percentile of a data set is the smallest value in the set with the property that *n*% of the data values are *less than or equal to* it. A data value that lies between two percentiles is said to lie in the lower percentile.

Time Out to Think

Is it possible for someone to score above the 100th percentile on a standardized test? Why or why not?

A *standard score table*, such as Table 6.3, allows us to find percentiles from standard scores. For each of many standard scores in a normal distribution, the table gives the percentage of values in the distribution *less than or equal to* that value. For example, Table 6.3 shows that 55.96% of the values in a normal distribution have a standard score less than or equal to 0.15. In other words, a data value with a standard score of 0.15 lies in the 55th percentile.

TABLE 6.3 Standard Scores and Percentiles

z-score	Percentile	*z*-score	Percentile	*z*-score	Percentile	*z*-score	Percentile
−3.5	0.02	−1.0	15.87	0.0	50.00	1.1	86.43
−3.0	0.13	−0.95	17.11	0.05	51.99	1.2	88.49
−2.9	0.19	−0.90	18.41	0.10	53.98	1.3	90.32
−2.8	0.26	−0.85	19.77	0.15	55.96	1.4	91.92
−2.7	0.35	−0.80	21.19	0.20	57.93	1.5	93.32
−2.6	0.47	−0.75	22.66	0.25	59.87	1.6	94.52
−2.5	0.62	−0.70	24.20	0.30	61.79	1.7	95.54
−2.4	0.82	−0.65	25.78	0.35	63.68	1.8	96.41
−2.3	1.07	−0.60	27.43	0.40	65.54	1.9	97.13
−2.2	1.39	−0.55	29.12	0.45	67.36	2.0	97.72
−2.1	1.79	−0.50	30.85	0.50	69.15	2.1	98.21
−2.0	2.28	−0.45	32.64	0.55	70.88	2.2	98.61
−1.9	2.87	−0.40	34.46	0.60	72.57	2.3	98.93
−1.8	3.59	−0.35	36.32	0.65	74.22	2.4	99.18
−1.7	4.46	−0.30	38.21	0.70	75.80	2.5	99.38
−1.6	5.48	−0.25	40.13	0.75	77.34	2.6	99.53
−1.5	6.68	−0.20	42.07	0.80	78.81	2.7	99.65
−1.4	8.08	−0.15	44.04	0.85	80.23	2.8	99.74
−1.3	9.68	−0.10	46.02	0.90	81.59	2.9	99.81
−1.2	11.51	−0.05	48.01	0.95	82.89	3.0	99.87
−1.1	13.57	0.0	50.00	1.0	84.13	3.5	99.98

QUICK QUIZ

Choose the best answer to each of the following questions. Explain your reasoning with one or more complete sentences.

1. Graphs of normal distributions
   a. always look exactly the same.
   b. always have the same characteristic bell shape.
   c. can have any shape as long as they have a sharp central peak.
2. In a normal distribution, the mean
   a. is equal to the median.
   b. is greater than the median.
   c. can be greater or less than the median.
3. In a normal distribution, data values farther from the mean are
   a. less common than data values close to the mean.
   b. more common than data values close to the mean.
   c. equally as common as data values close to the mean.
4. Consider wages at a fast-food restaurant where most of the workers earn the minimum wage. Would you expect the wages of all workers at this restaurant to have a normal distribution?
   a. Yes, because wage distributions are always normal.
   b. No, because the minimum wage is not enough money to live on.
   c. No, because the fact that no one earns less than the minimum wage means the distribution cannot be symmetric.
5. In a normal distribution, about 2/3 of the data values fall within
   a. 1 standard deviation of the mean.
   b. 2 standard deviations of the mean.
   c. 3 standard deviations of the mean.
6. Suppose a car driven under different conditions gets a mean gas mileage of 40 miles per gallon with a standard deviation of 3 miles per gallon. If you drive this car many times, your mileage on 99.7% of the trips will be between
   a. 37 and 43 miles per gallon.
   b. 40 and 43 miles per gallon.
   c. 31 and 49 miles per gallon.
7. Consider again the car described in Exercise 6. On about what percentage of the trips will your gas mileage be above 43 miles per gallon?
   a. 16%
   b. 2.5%
   c. 68%
8. Consider an exam with a normal distribution of scores with a mean of 75 and a standard deviation of 6. If you get an 84 on the exam, your *standard score* (z-score) is
   a. 84.
   b. 9.
   c. 1.5.
9. An acquaintance tells you that his IQ is in the 102nd percentile. You can conclude that
   a. he is smarter than 102% of all people.
   b. he is smarter than 2% of all people.
   c. he doesn't understand percentiles.
10. The height of a particular 7-year-old girl has a standard score of −0.60 among the heights of all 7-year-old girls. From Table 6.3, you know that
    a. she is 0.6 inch shorter than average for her age.
    b. she is taller than about 27% of all 7-year-old girls.
    c. she is shorter than about 27% of all 7-year-old girls.

Exercises

REVIEW QUESTIONS

1. What is a normal distribution? Briefly describe the conditions that make a normal distribution. Give an example of a data set that is likely to have a normal distribution, and explain why.
2. What is the 68-95-99.7 rule for normal distributions? Explain how it can be used to answer questions about frequencies of data values in a normal distribution.
3. What is a standard score? How do you find the standard score for a particular data value?
4. What is a percentile? Describe how Table 6.3 allows you to relate standard scores and percentiles.

DOES IT MAKE SENSE?

Decide whether each of the following statements makes sense (or is clearly true) or does not make sense (or is clearly false). Explain your reasoning.

5. The heights of male basketball players at Kentucky College are normally distributed with a mean of 6 feet 3 inches and a standard deviation of 3 inches.
6. The weights of babies born at Belmont Hospital are normally distributed with a mean of 6.8 pounds and a standard deviation of 1.2 pounds.

7. The weights of babies born at Belmont Hospital are normally distributed with a mean of 6.8 pounds and a standard deviation of 7 pounds.

8. On yesterday's mathematics exam, the standard score was 75.

9. My professor graded the final on a curve, and she gave a grade of A to anyone who had a standard score of 2 or more.

10. Jack is in the 50th percentile for height, so he is of median height.

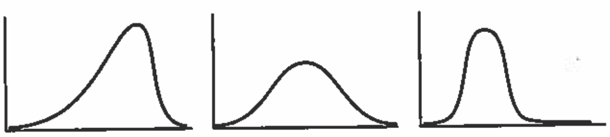
BASIC SKILLS & CONCEPTS

11–12: Normal Shape. Consider the following sets of three distributions, all of which are drawn to the same scale. Identify the two distributions that are normal. Of the two normal distributions, which one has the larger variation?

11.

(a) (b) (c)

12.



(a) (b) (c)

13–18: Normal Distributions. State, with an explanation, whether you would expect the following data sets to be normally distributed.

13. The delay in departure of trains from a station (note that trains, buses, and airplanes cannot leave early)

14. The weights of lawn fertilizer bags labeled "40 pounds"

15. The distances between the bull's-eye of a target and 100 darts thrown by an expert

16. The distances of tee shots hit with the same club by a professional golfer during a four-day tournament

17. Scores on an easy statistics exam

18. The times of runners at the Olympic marathon

19. **The 68-95-99.7 Rule.** A set of test scores is normally distributed with a mean of 100 and a standard deviation of 20. Use the 68-95-99.7 rule to find the percentage of scores in each of the following categories.

a. less than 100	b. less than 120
c. less than 140	d. less than 60
e. greater than 60	f. greater than 160
g. greater than 80	h. between 60 and 140

20. **The 68-95-99.7 Rule.** The resting heart rates for a sample of individuals are normally distributed with a mean of 70 and a standard deviation of 15. Use the 68-95-99.7 rule to find the percentage of heart rates in each of the following categories.

a. less than 55	b. less than 40
c. less than 85	d. less than 100
e. greater than 85	f. greater than 55
g. greater than 40	h. between 55 and 85

21–28: Standard Scores. The scores on a psychology exam were normally distributed with a mean of 67 and a standard deviation of 8.

21. About what percentage of scores were above 59?

22. About what percentage of scores were below 83?

23. A failing grade on the exam was anything 2 or more standard deviations below the mean. What was the cutoff for a failing score? Approximately what percentage of the students failed?

24. If 500 students took the exam, approximately how many students scored below 75?

25. What is the standard score for an exam score of 67?

26. What is the standard score for an exam score of 59?

27. What is the standard score for an exam score of 55?

28. What is the standard score for an exam score of 88?

29–30: Standard Scores and Percentiles. Use Table 6.3 to find the standard score and percentile of the following data values.

29. a. A data value 1 standard deviation above the mean
 b. A data value 0.5 standard deviation above the mean
 c. A data value 1.5 standard deviations below the mean

30. a. A data value 0.5 standard deviation below the mean
 b. A data value 2 standard deviations below the mean
 c. A data value 1.2 standard deviations above the mean

31–32: Percentiles. Use Table 6.3 to find the approximate standard score of the following data values. Then state the approximate number of standard deviations that the value lies above or below the mean.

31. a. A data value in the 20th percentile
 b. A data value in the 80th percentile
 c. A data value in the 63rd percentile

32. a. A data value in the 10th percentile
 b. A data value in the 35th percentile
 c. A data value in the 88th percentile

FURTHER APPLICATIONS

33–36: Pregnancy Length. Actual lengths of pregnancy terms are nearly normally distributed about a mean pregnancy length (of about 38 to 39 weeks) with a standard deviation of 15 days.

33. About what percentage of births would be expected to occur within 15 days of the mean pregnancy length?

34. About what percentage of births would be expected to occur within 1 month of the mean pregnancy length?

35. About what percentage of births would be expected to occur more than 15 days after the mean pregnancy length?