

BY THE WAY

Another way of interpreting the standard deviation uses a mathematical rule called *Chebyshev's theorem*. It states that, for any data set, at least 75% of all data values lie within 2 standard deviations of the mean, and at least 89% of all data values lie within 3 standard deviations of the mean.

BY THE WAY

Technologies such as catalytic converters have helped reduce the amounts of many pollutants emitted by cars (per mile driven), but increasing gas mileage is the only way to reduce the carbon dioxide emissions implicated in global warming. This is a major reason why auto manufacturers are developing high-mileage "hybrid" vehicles and zero-emission vehicles that run on electricity or fuel cells.



The range rule of thumb works reasonably well for data sets in which values are distributed fairly evenly. It does not work well when the high or low values are extreme outliers. You must use judgment in deciding whether the range rule of thumb is applicable in a particular case.

Example 4 Using the Range Rule of Thumb

Use the range rule of thumb to estimate the standard deviations for the waiting times at Big Bank and Best Bank. Compare the estimates to the actual standard deviations found in Example 3.

Solution The waiting times for Big Bank vary from 4.1 to 11.0 minutes, which means a range of $11.0 - 4.1 = 6.9$ minutes. The waiting times for Best Bank vary from 6.6 to 7.8 minutes, for a range of $7.8 - 6.6 = 1.2$ minutes. The range rule of thumb gives the following estimates for the standard deviations:

$$\text{Big Bank:} \quad \text{standard deviation} \approx \frac{6.9}{4} \approx 1.7$$

$$\text{Best Bank:} \quad \text{standard deviation} \approx \frac{1.2}{4} = 0.3$$

The actual standard deviations calculated in Example 3 are 1.96 and 0.44, respectively. For these two cases, the estimates from the range rule of thumb slightly underestimate the actual standard deviations. Nevertheless, the estimates put us in the right ballpark, showing that the rule is useful. **Now try Exercises 15–18, part d.**

Example 5 Estimating a Range

Studies of the gas mileage of a Prius under varying driving conditions show that it gets a mean of 45 miles per gallon with a standard deviation of 4 miles per gallon. Estimate the minimum and maximum gas mileage that you can expect under ordinary driving conditions.

Solution From the range rule of thumb, the low and high values for gas mileage are approximately

$$\begin{aligned} \text{low value} &\approx \text{mean} - 2 \times \text{standard deviation} \\ &= 45 - (2 \times 4) \\ &= 37 \end{aligned}$$

$$\begin{aligned} \text{high value} &\approx \text{mean} + 2 \times \text{standard deviation} \\ &= 45 + (2 \times 4) \\ &= 53 \end{aligned}$$

The range of gas mileage for the car is roughly from a minimum of 37 miles per gallon to a maximum of 53 miles per gallon. **Now try Exercises 15–18, part e.**

QUICK QUIZ

Choose the best answer to each of the following questions. Explain your reasoning with one or more complete sentences.

- The lowest score on an exam was 62, the median score was 75, and the high score was 96. The range was
 - 34.
 - 62.
 - 75.

- Which of the following is *not* part of a five-number summary?
 - the low value
 - the median
 - the mean

3. The lower quartile for wages at a coffee shop is \$8.25, and the upper quartile is \$10.75. What can you conclude?
 - a. Half the workers earn between \$8.25 and \$10.75.
 - b. The median is \$9.50.
 - c. The range is \$2.50.
4. Is it possible for a distribution to have a mean that is higher than its upper quartile?
 - a. Yes; this is always the case.
 - b. Yes, but only if there are outliers at high values.
 - c. No.
5. Suppose you are given the mean and just one data value from a distribution. What can you calculate?
 - a. the range
 - b. the deviation for the single data value
 - c. the standard deviation
6. The standard deviation is best described as a measure of
 - a. the average values in a data set.
 - b. the spread of the data values around the mean.
 - c. the range of the data values.
7. What type of data distribution has a *negative* standard deviation?
 - a. distribution in which most of the values are negative numbers
 - b. distribution in which most of the values lie below the mean
 - c. none—the standard deviation cannot be negative
8. In any distribution, it is always true that
 - a. the range is at least as large as the standard deviation.
 - b. the standard deviation is at least as large as the range.
 - c. the range is always at least twice the mean.
9. Which data set would you expect to have the highest standard deviation?
 - a. heights (lengths) of newborn infants
 - b. heights of all elementary school children
 - c. heights of first-grade boys
10. Professors Smith, Jones, and Garcia all got the same mean grade of 2.7 (a B-) on their student evaluations last semester. The standard deviations were 0.2 for Smith, 0.5 for Jones, and 1.1 for Garcia. Which professor received a very high grade from the most students?
 - a. Smith
 - b. Jones
 - c. Garcia

Exercises

REVIEW QUESTIONS

1. Consider two grocery stores at which the mean time in line is the same but the variation is different. At which store would you expect the customers to have more complaints about the waiting time? Explain.
2. Describe how we define and calculate the range of a distribution.
3. What are the quartiles of a distribution? How do we find them?
4. Define the five-number summary, and explain how to depict it visually with a boxplot.
5. Describe the process of calculating a standard deviation. Give a simple example of its calculation (such as calculating the standard deviation of the numbers 2, 3, 4, 4, and 6). What is the standard deviation if all of the sample values are the same?
6. Briefly describe the use of the range rule of thumb for interpreting the standard deviation. What are its limitations?
8. The highest exam score was in the upper quartile of the distribution.
9. For the 30 students who took the test, the high score was 80, the median was 74, and the low score was 40.
10. I examined the data carefully, and the range was greater than the standard deviation.
11. The standard deviation for the heights of a group of 5-year-old children is smaller than the standard deviation for the heights of a group of children who range in age from 3 to 15.
12. The mean gas mileage of the compact cars we tested was 34 miles per gallon, with a standard deviation of 5 gallons.

BASIC SKILLS & CONCEPTS

13. **Big Bank Verification.** Find the mean and median for the waiting times at Big Bank given in the beginning of this unit. Show your work clearly, and verify that both are 7.2 minutes.
14. **Best Bank Verification.** Find the mean and median for the waiting times at Best Bank given in the beginning of this unit. Show your work clearly, and verify that both are 7.2 minutes.
- 15–18: **Comparing Variations.** Consider the following data sets.
 - a. Find the mean, median, and range for each of the two data sets.
 - b. Give the five-number summary and draw a boxplot for each of the two data sets.
 - c. Find the standard deviation for each of the two data sets.

DOES IT MAKE SENSE?

Decide whether each of the following statements makes sense (or is clearly true) or does not make sense (or is clearly false). Explain your reasoning.

7. Both exams had the same range, so they must have had the same median.

d. Apply the range rule of thumb to estimate the standard deviation of each of the two data sets. How well does the rule work in each case? Briefly discuss why it does or does not work well.

e. Based on all your results, compare and discuss the two data sets in terms of their center and variation.

15. The table below gives the cost of living index for six East Coast cities and six West Coast cities (using the ACCRA index, where 100 represents the average cost of living for all participating cities with a population of over 1.5 million).

East Coast Cities		West Coast Cities	
Atlanta	98.2	Los Angeles	155.8
Baltimore	108.7	Portland	113.2
Boston	135.4	San Diego	144.8
Miami	111.5	San Francisco	182.4
New York City	216.0	San Jose	156.0
Washington, DC	140.0	Seattle	122.7

16. The table below gives the average state and local taxes paid by a family of four with an income of \$75,000 in ten (east and west) coast cities and ten noncoastal cities.

East/West Coast Cities		Noncoastal Cities	
Atlanta	8261	Albuquerque	6369
Baltimore	8923	Cheyenne	2899
Boston	7875	Columbus, OH	7434
Los Angeles	7488	Denver	5535
New York City	10,490	Detroit	8257
Portland	7789	Kansas City	6805
Providence	9729	Louisville	7954
Seattle	4876	Oklahoma City	6593
Virginia Beach	6411	Salt Lake City	7153
Washington, DC	7594	Wichita	5683

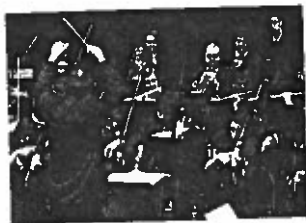
Source: District of Columbia Government.

17. Researchers at the Pennsylvania State University conducted experiments with popular trees in which one group of trees was given fertilizer and irrigation while the other group was given no treatment. The weight in kilograms of trees in the two groups are as follows:

No treatment:	0.15	0.02	0.16	0.37	0.22
Treatment:	2.03	0.27	0.92	1.07	2.38

18. The following data sets give the approximate lengths of Beethoven's nine symphonies and Mahler's nine symphonies (in minutes).

Beethoven:	28	36	50	33	30	40	38	26	68
Mahler:	52	85	94	50	72	72	80	90	80



FURTHER APPLICATIONS

19–25: Understanding Variation. The following exercises give four data sets consisting of seven numbers.

- Make a histogram for each set.
- Give the five-number summary and draw a boxplot for each set.
- Compute the standard deviation for each set.
- Based on your results, briefly explain how the standard deviation provides a useful single-number summary of the variation in these data sets.

19. The following sets of numbers all have a mean of 9:

$$\{9, 9, 9, 9, 9, 9, 9\}, \{8, 8, 9, 9, 9, 10, 10\},$$

$$\{8, 8, 8, 9, 10, 10, 10\}, \{6, 6, 6, 9, 12, 12, 12\}$$

20. The following sets of numbers all have a mean of 6:

$$\{6, 6, 6, 6, 6, 6, 6\}, \{5, 5, 6, 6, 6, 7, 7\},$$

$$\{5, 5, 5, 6, 7, 7, 7\}, \{3, 3, 3, 6, 9, 9, 9\}$$

21. **Pizza Deliveries.** After recording the pizza delivery times for two different pizza shops, you conclude that one pizza shop has a mean delivery time of 45 minutes with a standard deviation of 3 minutes. The other shop has a mean delivery time of 42 minutes with a standard deviation of 20 minutes. Interpret these figures. If you liked the pizzas from both shops equally well, which one would you order from? Why?

22. **Airline Arrival Times.** Two airlines have data on the arrival times of their flights. An arrival time of +2 minutes means the flight arrived 2 minutes early. An arrival time of -5 minutes means the flight arrived 5 minutes late. Skyview Airlines has a mean arrival time of 0.5 minute with a standard deviation of 9.6 minutes. SkyHigh Airlines has a mean arrival time of -2.5 minutes with a standard deviation of 4.0 minutes. Explain the meaning of these figures and why they would affect your choice of airlines.

23. **Portfolio Standard Deviation.** The book *Investments* by Zvi Bodie, Alex Kane, and Alan Marcus claims that the returns for investment portfolios with a single stock have a standard deviation of 0.55, while the returns for portfolios with 32 stocks have a standard deviation of 0.325. Explain how the standard deviation measures the risk in these two types of portfolios.

24. **Defect Rates.** Two factories each produce 1000 computer chips per day. In Factory A, the number of defective chips per day is 3 with a standard deviation of 2.5. In Factory B, the number of defective chips per day is 4 with a standard deviation of 0.5. Use these figures to either defend or discredit the claim that Factory A has a more reliable manufacturing process.

25. **Batting Standard Deviation.** For the last 100 years, the mean batting average in the major leagues has remained fairly constant at about .260. However, the standard deviation of batting averages has decreased from about .049 in the 1870s to .031 at present. What does this tell us about the batting averages of players? Based on these facts, would you expect batting averages above .350 to be more or less common today than in the past? Explain.